Network Motif-Inspired Evolution of Hodgkin-Huxley Neuronal Networks with Spike-Timing Dependent Plasticity

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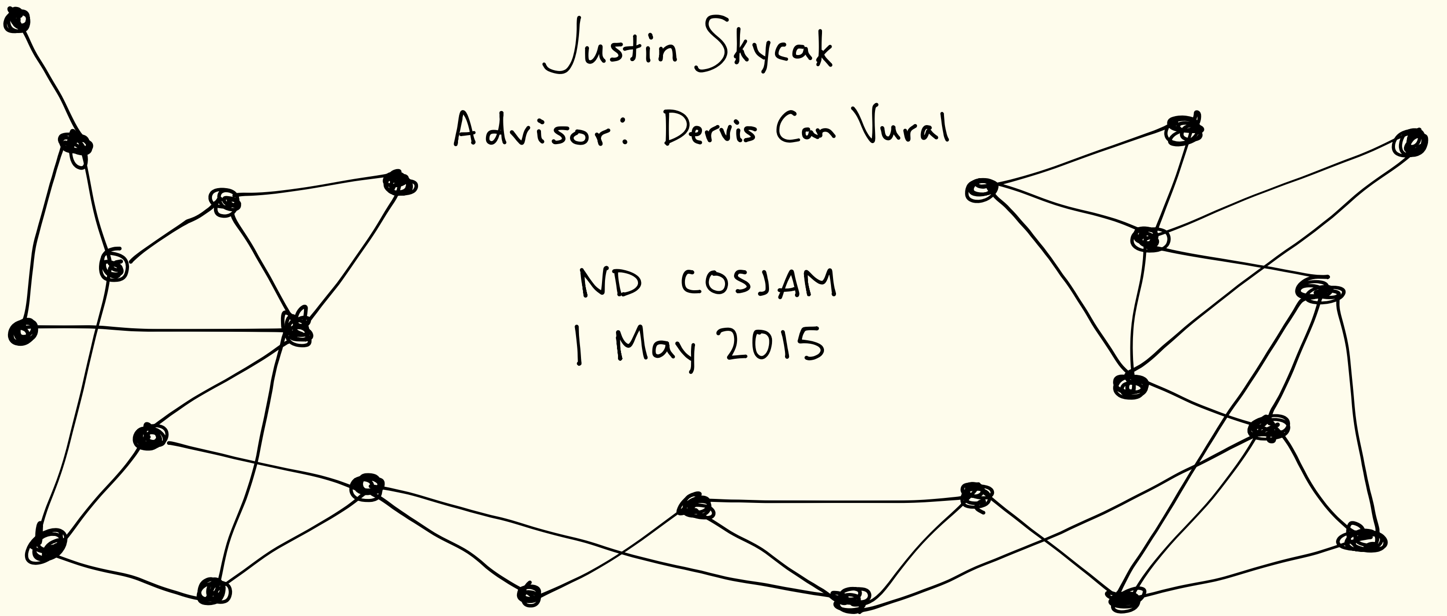
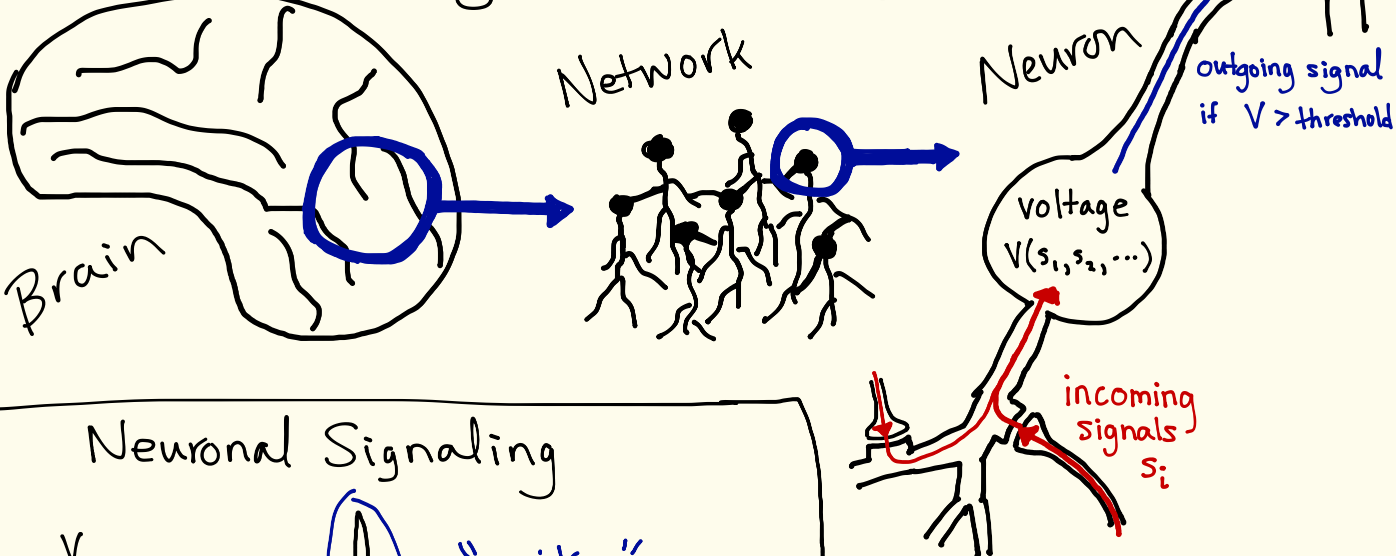
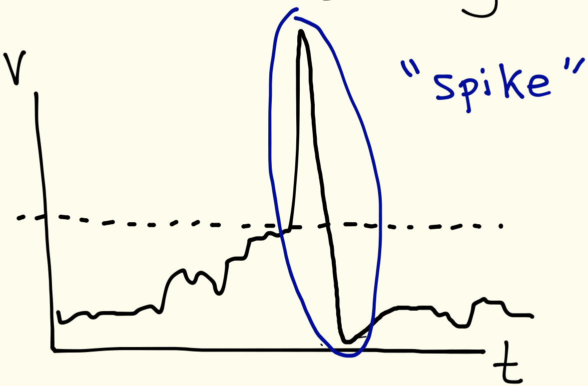


The Basics



Neuronal Signaling

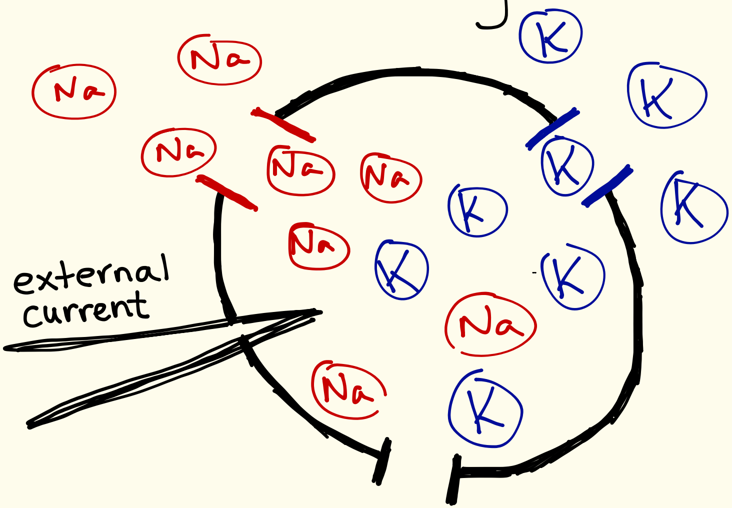


very simplified picture

Hodgkin-Huxley model

$$C\frac{dV}{dt} = \text{external current} - \sum_{\text{Na, K, leak}} \text{ion channel currents}$$

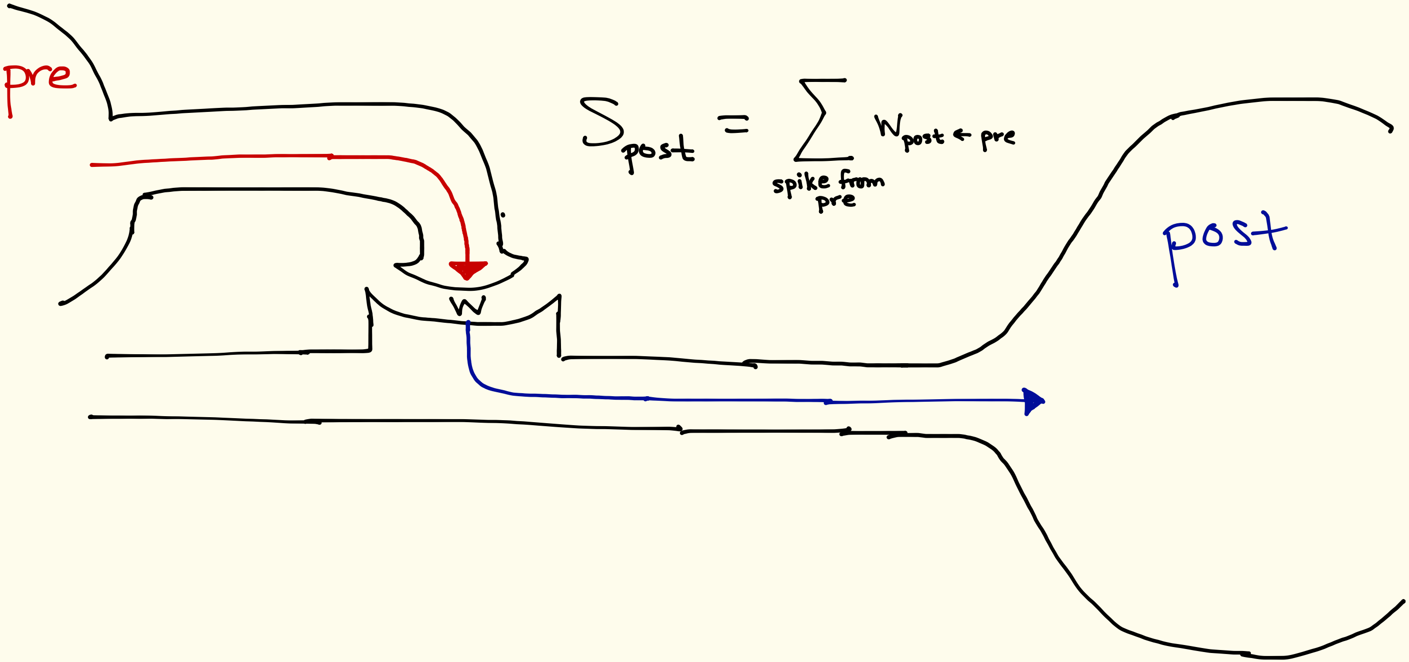
$$\text{ion channel current} = g \cdot (\text{activation}) \cdot (\text{inactivation}) \cdot (V - \text{reversal potential})$$



- Reproduces spikes
- Include additional ion channels for specialized functions

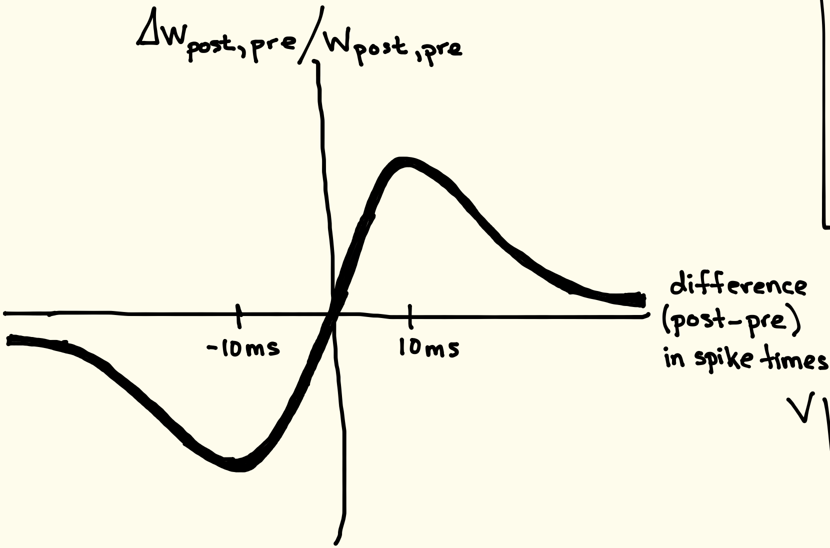
Connections

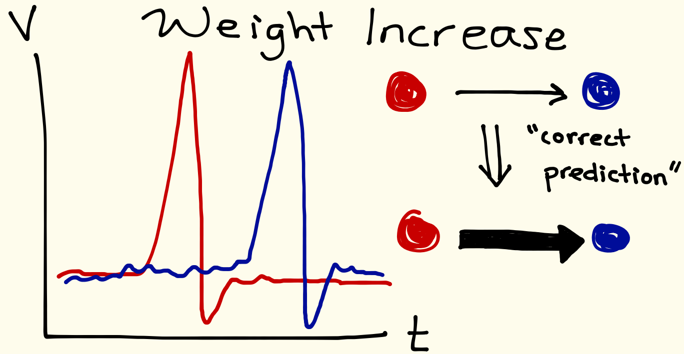
* when pre spikes, stimulate post with current $w_{post \leftarrow pre}$ *

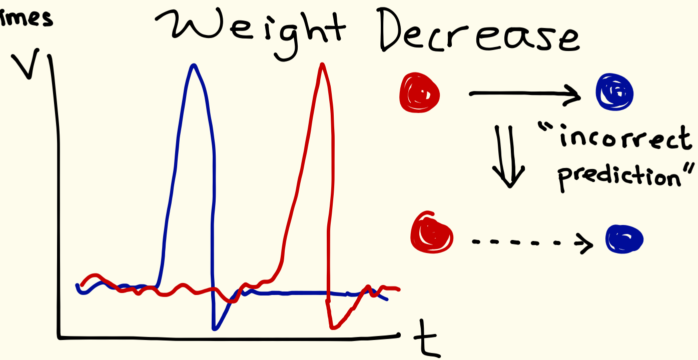


$$S_{post} = \sum_{\text{spike from pre}} w_{post \leftarrow pre}$$

Spike-Timing Dependent Plasticity (STDP)



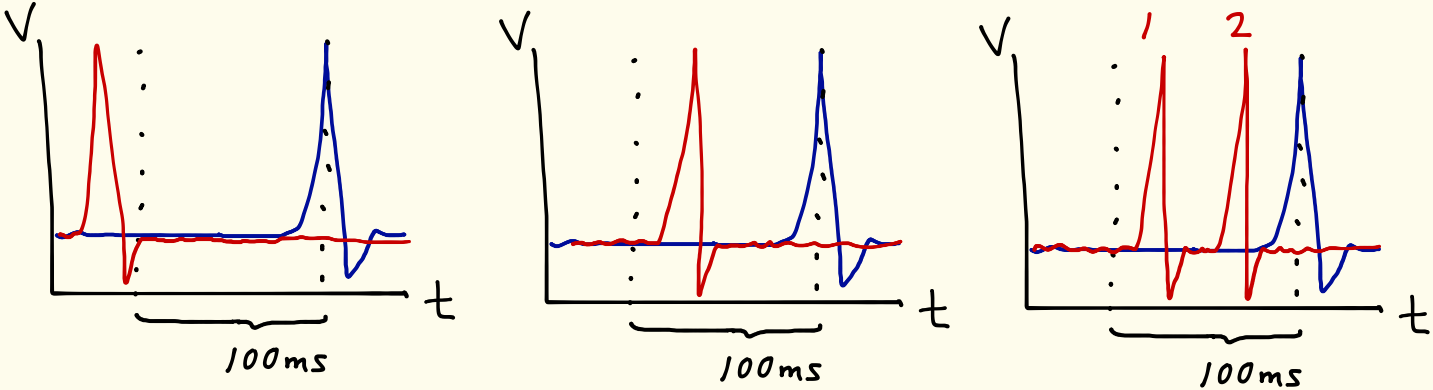




form: $\gamma \times e^{-x/10ms}$

↑ learning rate

Nearest-neighbor within 100 ms



ignore spike

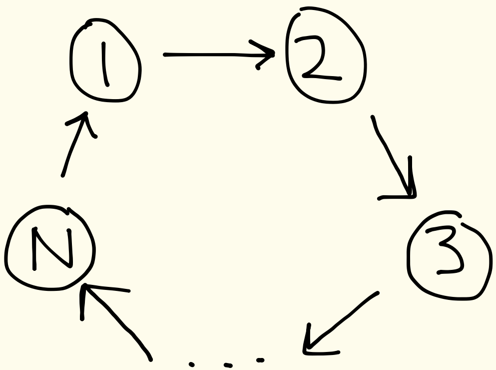
use spike

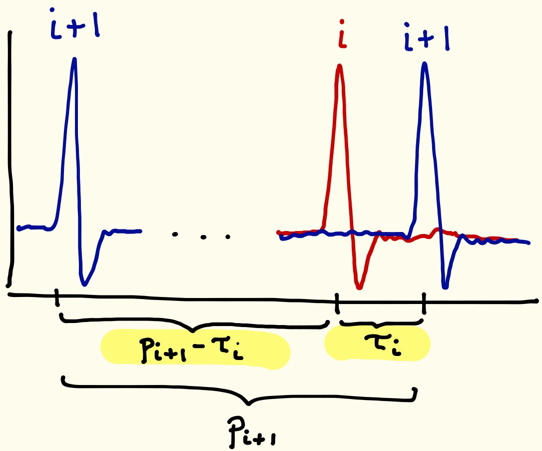
use spike 2 only

Cycles

p_i : period (ms) at which neuron i spikes

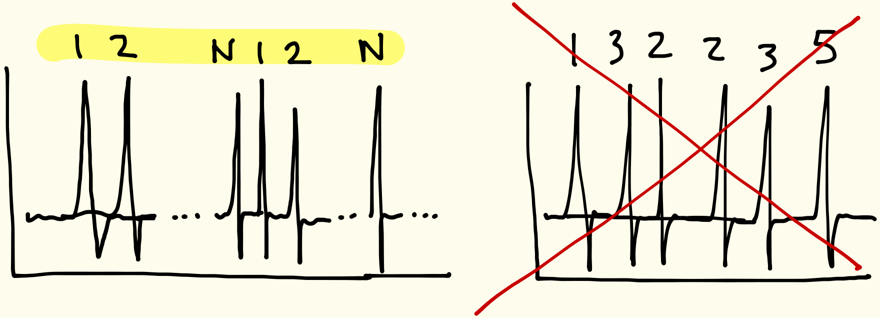
τ_i : time (ms) it takes for neuron $i+1$ to fire after neuron i has fired





STDP in cycles

Assume sequential spiking.



Then under STDP,

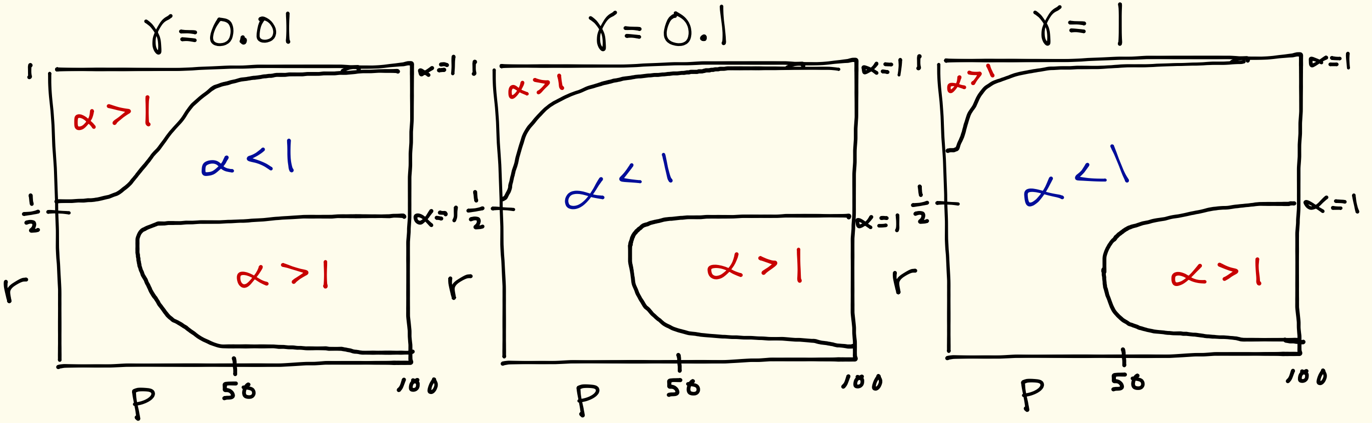
$$w_{i+1\leftarrow i} \xrightarrow{\text{reward}} w_{i+1\leftarrow i}\left(1+\gamma\tau_i e^{-\tau_i/10}\right)$$

$$w_{i+1\leftarrow i} \xrightarrow{\text{penalty}} w_{i+1\leftarrow i}\left[1-\gamma\left(p_{i+1}-\tau_i\right)e^{-(p_{i+1}-\tau_i)/10}\right]$$

- Abuse of notation: drop subscripts (should be clear)
- With each round through the spike cycle,

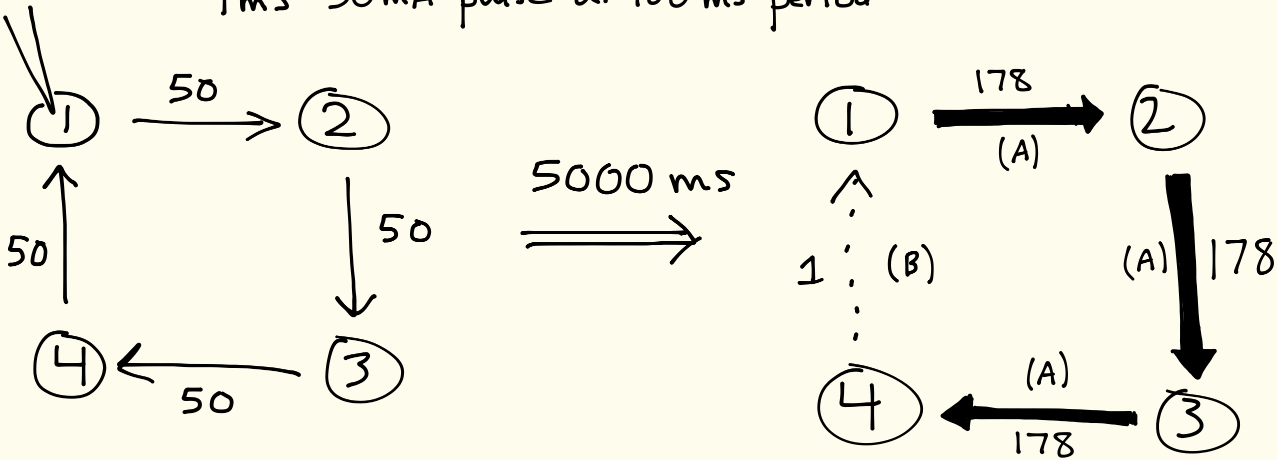
$$w \to w\underbrace{\left(1+\gamma\tau e^{-\tau/10}\right)\left(1-\gamma(p-\tau)e^{-(p-\tau)/10}\right)}_{\alpha}$$

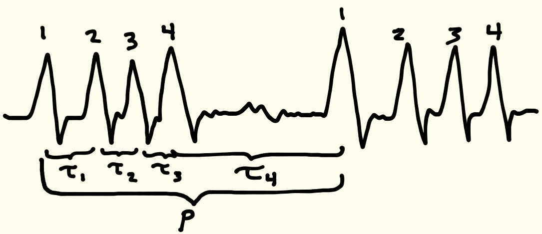
- Let $0<r=\tau/p<1$. Then $\alpha = 1+\gamma r p e^{-rp/10} - \gamma(1-r)pe^{-(1-r)p/10} - \gamma^2 r(1-r)p^2 e^{-p/10}$

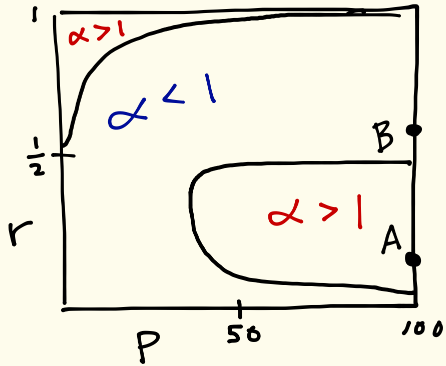


Simulation ($\gamma = 0.1$)

1ms 50mA pulse at 100ms period





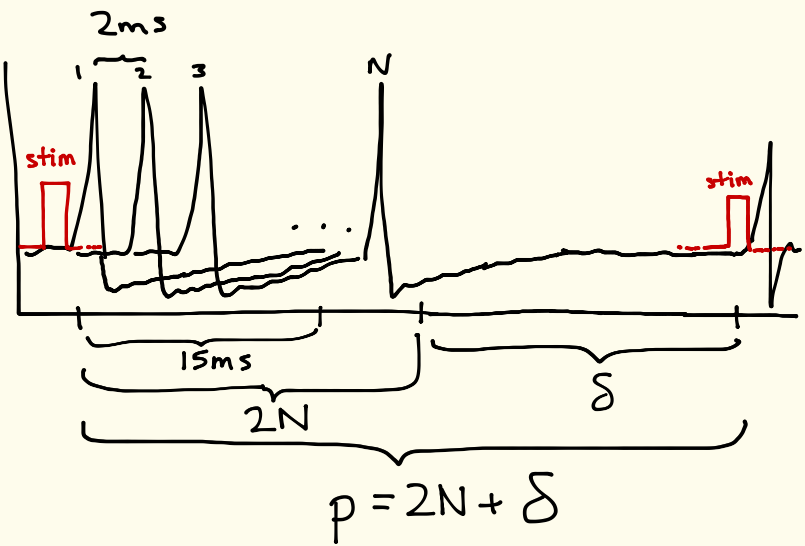


When does sequential spiking hold?

in simulation,

absolute refractory period ≈ 3ms
+ relative refractory period ≈ 12 ms

total refractory period ≈ 15 ms

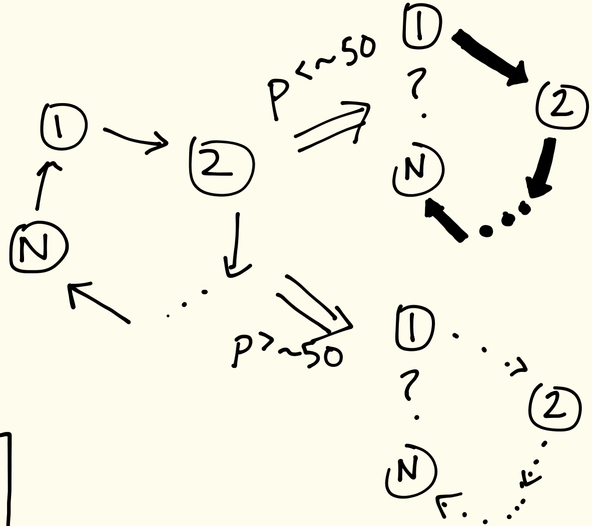


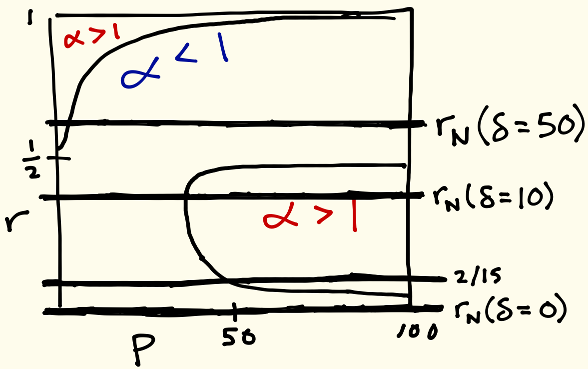
to maintain seq. spiking:

- $2N < 15 \rightarrow$ any δ
  (although $\delta < 15 - 2N$ may cause $p >$ stimulus period due to ① refractory period)
- $2N > 15 \rightarrow$ Ⓝ causes ① spike
  → self-sustaining cycle
  → $\delta = 0$ or single pulse stimulus

Small cycle ($N < \sim 7$)

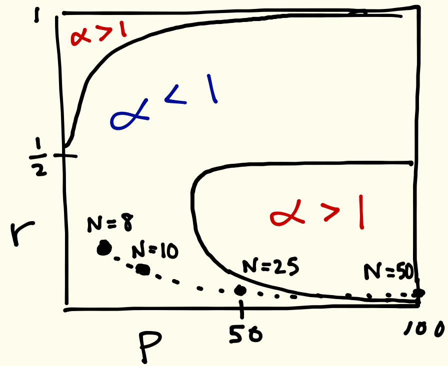
- Must choose $\delta \geq 15 - 2N$.
- $r = \dfrac{\tau}{15+\delta} \leq \dfrac{\tau}{15}$
- $r_1 \approx r_2 \approx \cdots \approx r_{N-1} \approx \dfrac{2}{15}$
- $r_N \approx \dfrac{\delta}{15+\delta}$

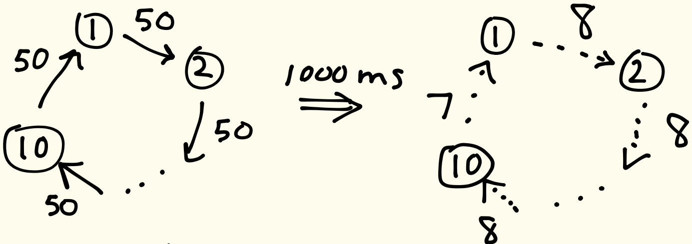


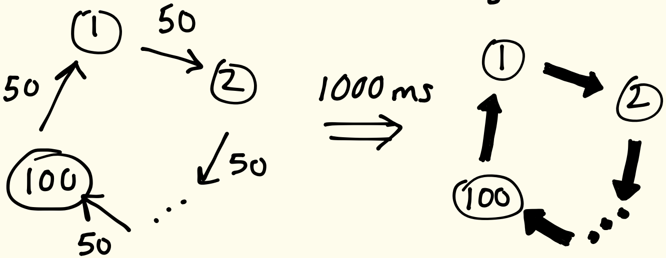


Large cycle ($N > \sim 7$)

- $\delta = 0$ or single-pulse stimulus. Either way, $p \approx 2N$.
- $r_1 \approx r_2 \approx \cdots \approx r_N \approx \dfrac{2}{2N} = \dfrac{1}{N}$





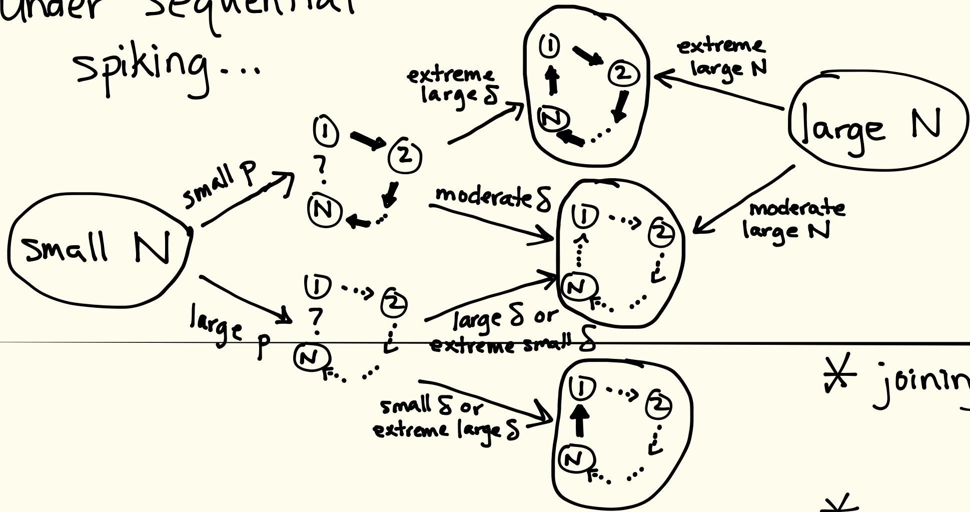


all weights between 100 and 110

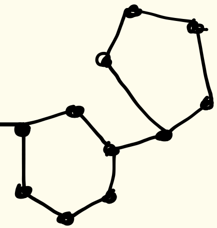
(single 1ms 50 mA pulse for both simulations)

Conclusions and Future Directions

under sequential spiking...



* more precise quantification
* more verification (untested cases)
* joining cycles:



* nonsequential Spiking?